Marshall Clagett, *Archimedes in the Middle Ages*. Volume V. Quasi-Archimedean Geometry in the Thirteenth Century. In two books. (Memoirs of the American Philosophical Society, Volume 157 A+B). Philadelphia: American Philosophical Society, 1984. ix+716 pp. $ 50.00.

Published in ***Annals of Science*** **46** (1989), 107–109.

Volume V of Marshall Clagett's monumental *Archimedes in the Middle Ages* is a supplement dealing with thirteenth century geometrical works with 'overtones of Archimedean geometry' (p. 147), and is in itself a monumental work. Its main part contains very careful critical editions of Gerard of Brussels' *Liber de Motu*, of Jordanus de Nemore's *Liber Philotegni*, and of the *Liber de triangulis Jordani*, which is argued convincingly *not* to have been written by Jordanus. One of the appendices presents an edition of John Dee's *Inventa* on the parabola. All of these editions are followed by equally careful translations into English, and provided with illuminative and extensive but non-pedantic mathematical and mathematico-historical commentaries, amply cross-referenced to earlier volumes. Book 2 contains the diagrams (with indication of errors in and major mutual disagreement between diagrams in different manuscripts), together with the bibliography and four indices: One of Latin Mathematical Terms, one of Manuscripts Cited, one of Citations of Euclid's *Elements*, and one of Names and Works.

Clagett edited the *Liber de motu* for the first time in 1956[1], interpreting the work at that time primarily as a kinematic treatise (the way Bradwardine was to read and comment upon it). The new edition is motivated in part by the inclusion of Richard de Fournival's manuscript of the work, in part by a better understanding of intricate central points and by a reading of the whole work as being primarily a piece of geometrical theory, dealing with the average movement of lines, surfaces and bodies when rotated about an axis. The average movement of a line can of course be connected intuitively to the surface described, and that of a plane figure around a line in the same plane to that of the volume described; this is also what Gerard does. In order to visualize the average movement of a circle rotated around its centre, Gerard replaces it with the right triangle of area $r \cdot 2\pi r$ raised perpendicularly to the plane of the circle and standing on a radius (the essential trick missed in earlier interpretations of the text); this again permits him to regard a body with rotational symmetry rotated around its axis as composed of circles, the average movements of which can be calculated separately. Gerard's method is thus spiritually related to that of Archimedes' *Method*, and to those later practiced by Galileo and Cavalieri.

The edition of the *Liber philotegni* turns out to be a first edition, since the work proves to be different from the *Liber de triangulis Jordani* edited by Curtze[2]. Its 64 propositions[3] deal with a variety of geometrical problems, including the partition of triangles, comparisons of angles versus sides in polygons and of angles versus chords in circles and pairs of tangent circles, and the comparison or mutual inscription / circumscription of polygons. The central (and interrelated) topics are the comparison of polygons and what Clagett labels 'geometric trigonometry'. As it is to be expected from Jordanus, the treatise is a sophisticated and highly original piece of work incorporating a variety of earlier problems and results into a fairly coherent whole. Some of these influences are discussed in the mathematical commentary and in Appendix III. One which goes unmentioned is an interest shared with the 10th-century Islamic mathematician Abū'l-Wafāʾ, *viz.* an interest in regular polygons inscribed in other regular polygons[4]; however, since no single construction is shared with Abū'l-Wafāʾ's elementary treatise, Jordanus' source for the idea must presumably be some unknown translation from an unidentified Islamic work.

Curtze's edition of the *Liber de triangulis Jordani* was made from a single manuscript, and not always very correctly. It is thus completely superseded by Clagett's new edition. As mentioned, the work is different from the *Liber philotegni*, and written by a different person, as shown by Clagett in his analysis. Certainly, the basis is the *Liber philotegni* – that is, a version containing only propositions 1-46 and '46+1'; the changes, however, are manifold. Elementary auxiliary propositions are inserted; proofs are different and often given in outline only; the work is structured in 4 books (corresponding to material subdivisions of the *Liber philotegni*); in the end of the work, a number of extra propositions, many of them taken over almost literally from other sources (cited in appendix III in as far as identified and not edited in Vol. I of Clagett's work) make up the final and greater part of book IV. These differences are all discussed by Clagett. A final difference *not* discussed is a definitely oral flavour evident until prop. IV.16. This stylistic peculiarity is one of several reasons why I doubt Clagett's assumption that the work is due to a later author trying to prepare an 'improved' version from a Jordanian manuscript; as I have argued in some depth elsewhere[5], the work is rather a students' *reportatio* of a series of lectures held over *Liber philotegni* at a time when this work had only grown to 46+1 propositions (and thus either held by Jordanus himself or at least when he was still at work) – the appended final propositions being presumably copied from material not covered during the lectures but put at the student's disposal by the lecturer (an erroneous second part of prop. IV.13 being perhaps due to the student's own limited genius).

The edition of John Dee's *Inventa* from the autograph is a continuation of Vol. IV of Clagett's work, which deals with the Medieval traditions of conic sections. It is not very impressing as far as mathematical substance is concerned, but all the more interesting as evidence of Dee's mathematical level and ideals, and thus of that context in which he was respected and influential. Dee makes use of formals proofs – but not systematically, and not concerning the more delicate properties of the parabola (where he tacitly presupposes current standard works on the subject). He depends on the Medieval traditions and not on Apollonios, but differs from the Medieval predecessors in several ways: He expands the number of definitions to 49 (mostly trivially); a large part of his propositions resemble the *Data*-type ('when entities *a*, *b*, ... are given, entity *p* will also be given'), but interprets this pattern 'algebraically' ('when entities a, b, ... are given, how to find entity *p*'); he disregards the traditional distinction between geometric construction and numerical calculation, and goes further than even Regiomontanus (as edited in Vol. IV) toward the algebraization of the subject and the expression of results in tabular form, expressing the numerical procedures in the traditional language of Medieval algebra and referring repeatedly to the sine table. As Clagett concludes (p. 493), Dee shows himself to be less original *as a mathematician* (my emphasis – JH) than Regiomontanus and Werner, but more so than Oronce Fine. Yet better than any of these he expresses the preparation of the ground for Descartes' synthesis of geometry and algebraic analysis (another example, we might say, of 16th century occultist utilitarianism anticipating but not realizing ideas of 17th century science). Like his older contemporaries Cardano and Stifel, Dee is an exponent of the environment of mathematical practice turning to theory rather than an instance of 'academic' theory orienting itself toward practical goals.

To sum up, Clagett has provided us with excellent editions of some very important texts, including the most original geometrical works of the Latin 13th century, and he has done much to establish the connections between these texts and those inspiring or inspired by them. At the same time, he has carefully avoided any pretense to exhaust the historical implications of the texts presented; we owe him our thanks for having created the basis for raising new questions and for confronting old problems anew.

Printing, proof-reading, paper quality and binding are of the fine quality known from earlier volumes of the work.

Jens Høyrup

NOTES

1. M. Clagett, 'The *Liber de motu* of Gerard of Brussels and the Origins of Kinematics in the West', *Osiris*, **12** (1956), 73-175.

2. M. Curtze (ed.), 'Jordani Nemorarii Geometria vel De triangulis libri IV', *Mitteilungen des Coppernicusvereins für Wissenschaft und Kunst zu Thorn*, **6** (Thorn / Torun ├'┤, 1887).

3. One of the two manuscripts containing the full work has only 63, but announces 64 in its headline; in the reviewer's opinion, the proposition labelled '46+1' by Clagett (which was also taken over in the *Liber de triangulis)* was probably forgotten by the rather careless scribe.

4. See chapter 7 of Abū'l-Wafāʾ's *Book on What is Necessary from Geometric Construction for the Artisan* – pp. 85-93 in S. A. Krasnova (ed., tr.), 'Abu-l-Vafa al-Buzdžani, *Kniga o tom, čto neobxodimo remeslenniku iz geometričeskix postroenij', in* A. T. Grigor'jan & A. P. Juškevič (eds), *Fiziko-matematičeskie nauki v stranax vostoka.* Sbornik statej i publikacij. 4 vols (Moscow, 1966), I, 42-140.

5. Chapter IX in H¢yrup, 'Jordanus de Nemore, 13th Century Mathematical Innovator: an Essay on Intellectual Context, Achievement, and Failure', *Archive for History of Exact*

Sciences, in press.